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AXISYMMETRIC STATIC NETS

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Abstract—A class of nets is studied which are statically indeterminate and may possess a unique configuration despite their high degree of kinematic indeterminacy. The intricately interrelated statical and geometric properties of such nets are revealed and investigated on the basis of closed-form solutions obtained for several main types of axisymmetric nets. A special emphasis is placed on the important case of torque-free nets. Together with a previously published paper[8] this study constitutes a compendium of results for axisymmetric static nets.

INTRODUCTION

A net represents a general-chain multidegree of freedom system[1–4] with intricately interrelated statics and geometry. The equilibrium configuration of a net is determined by the applied load. Conversely, for any given configuration, equilibrium is possible only under certain types of loads (equilibrium loads). However, the internal force distribution in a net under an equilibrium load may be non-unique. This exceptional case occurs when the homogeneous system of equations of statics of the net allows a non-trivial solution. In such a configuration the net is statically indeterminate while retaining its kinematic indeterminacy as well. Furthermore, the above non-trivial solution indicates, at least formally, the possibility of initial forces (prestress) in the net. If these forces are tensile, the state of prestress is stable and the net completely lacks mobility; it possesses a unique configuration in spite of its numerous degrees of freedom.

Thus, the significance of the described exceptional nets (referred to as static nets) is the uniqueness of their configurations or, at least, their statical indeterminacy which enables the most favorable of the statically permissible states to be chosen and realized.

The formal characteristic sign of a static net is the existence of a non-trivial solution to the homogeneous system of equations of statics. To retain the possibility of choosing a suitable coordinate system later, these equations are presented in an invariant vectorial form[5]:

$$\begin{aligned} (T_{u,i} - T_u R_i)u^i \sin\omega - T_u\kappa_u \cos\omega - T_v\kappa_v &= 0 \\ (T_{v,i} - T_v R_i)v^i \sin\omega + T_v\kappa_v \cos\omega + T_u\kappa_u &= 0 \\ T_u\sigma_u + T_v\sigma_v &= 0. \end{aligned} \tag{1}$$

Here u^i and v^i are the direction unit vectors of the net lines; T_u and T_v are the net forces referred to unit width strips $ds_v = 1$ and $ds_u = 1$, respectively; ω is the net angle; κ and σ are, respectively, the geodesic and normal curvatures of the net line u or v, according to the subscript; and finally,

$$R_i = \frac{1}{\sin^2\omega}[\lambda_u(u_i - v_i\cos\omega) + \lambda_v(v_i - u_i\cos\omega)], \tag{2}$$

where λ_u and λ_v are the Chebyshev curvatures of the net lines to be introduced later along with the geodesic and normal curvatures. The index $i = 1, 2$ denotes either the vector components or the partial derivatives of scalars using the summation convention for a repeating index.

For a given net with all the necessary geometric parameters fixed, system (1) is overdetermined (3 equations in 2 unknown forces). Except for an asymptotic net ($\sigma_u = \sigma_v = 0$), this system is compatible if and only if the following integrability condition is satisfied:

$$S_k \equiv \frac{1}{\sin^2 \omega}\left[\left(\frac{\sigma_v}{\sigma_u}\kappa_v - \kappa_u \cos \omega\right) u_k + \left(\frac{\sigma_u}{\sigma_v}\kappa_v - \kappa_u \cos \omega\right) v_k\right]$$
$$-\frac{1}{\sin \omega}\left(\frac{\sigma_{u,i}}{\sigma_u} v^i u_k - \frac{\sigma_{v,i}}{\sigma_v} u^i v_k\right) + R_k = \text{grad}. \tag{3}$$

This means that the vector S_k (called the static vector) with components (3) depending only on the geometric parameters of the net, must be the gradient of some scalar function S (static potential). If this is the case, system (1) allows the following solution in terms of the static potential:

$$T_u = C\sigma_v \exp S, \qquad T_v = -C\sigma_u \exp S, \tag{4}$$

where C is an arbitrary constant determining the magnitude of the initial forces.

The integrability condition (3) plays an important role in the synthesis of static nets. By imposing some special requirements on the net sought but not describing it completely the condition can be used as an equation from which the unspecified geometric parameters are determined so as to render the net of interest static. For axisymmetric nets an ordinary differential equation results so that the closed-form solutions are obtainable in many cases. Note that suitable segments of axisymmetric nets can be used for applications where axial symmetry is irrelevant, such as cable roofs.

In what follows, several main types of axisymmetric static nets (generally, not possessing reflection symmetry relative to a meridional plane) are identified and investigated. Differential geometry relations which can be found in the literature[6, 7] are presented here without development.

CHEBYSHEV NETS: GEOMETRY

This is one of the most common nets: all of its elementary cells are rhombi. It was proved by Chebshev that due to and at the expenxe of the varying net angle, the net is applicable to any smooth surface and is determined to within two arbitrary functions of one variable. It is convenient at first to identify a general axisymmetric net by the angles, α and β, (Fig. 1), that the net lines form with a meridian on a surface of revolution. If the surface is referred to cylindrical coordinates (r, ϕ, z), both α and β, as well as the net angle, $\omega = \beta - \alpha$, are functions of z alone and the direction unit vectors of the net lines are

$$u_1 = -A \sin \alpha, \qquad u^1 = \frac{\cos \alpha}{A}, \qquad u_2 = B \cos \alpha, \qquad u^2 = \frac{\sin \alpha}{B}, \tag{5}$$

v_i and v^i being obtainable by replacing α with β. The chosen Lamé parameters of the surface

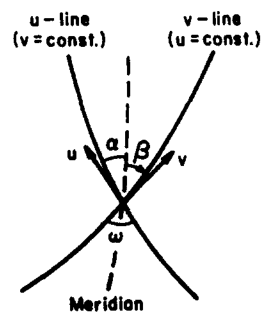

Fig. 1. Direction unit vectors of the net.

are

$$A = \sqrt{(1 + r'^2)}, \qquad B = r, \tag{6}$$

where the prime denotes differentiation with respect to z. With this choice, the Chebyshev curvatures of an axisymmetric net are given by

$$\lambda_u = \frac{r'}{rA}\sin\alpha + \frac{\beta'}{A}\cos\alpha, \qquad \lambda_v = \frac{r'}{rA}\sin\beta + \frac{\alpha'}{A}\cos\beta. \tag{7}$$

A Chebyshev net is characterized by both λ_u and λ_v becoming zero which results in

$$\frac{r'}{r} = -\alpha'\,\mathrm{ctn}\,\beta, \qquad \frac{r'}{r} = -\beta'\,\mathrm{ctn}\,\alpha. \tag{8}$$

Solving these equations simultaneously yields

$$r\frac{\cos\beta}{\sin\omega} = a, \qquad r\frac{\cos\alpha}{\sin\omega} = b, \tag{9}$$

where a and b are arbitrary constants. Formulas (9) play the same key role for a Chebyshev net as do the Clairaut formulas[6] for a geodesic net[8].

To establish some global properties of an axisymmetric Chebyshev net let $b > a$ and $\beta > 0$. Then, according to (9), $\beta > |\alpha|$,

$$\frac{\cos\alpha}{\cos\beta} = \frac{b}{a} \tag{10}$$

and

$$r = b\sin\beta - a\sin\alpha. \tag{11}$$

From here it can be easily found that, consistent with the above inequalities, the minimum and the maximum radii of the net are, respectively,

$$r_1 = r|_{\alpha=\beta=\pi/2} = b - a, \qquad r_2 = r|_{\alpha=-\beta=-\pi/2} = b + a. \tag{12}$$

Another important radius obtainable from (10) and (11) is

$$r_0 = r|_{\alpha=0} = b\sin\left(\arccos\frac{a}{b}\right) = \sqrt{(b^2 - a^2)}, = \sqrt{(r_1 r_2)}. \tag{13}$$

Resolving now eqns (9) with respect to α and β allows the result to be presented as

$$\sin\alpha = \frac{r_0^2 - r^2}{2ar}, \qquad \sin\beta = \frac{r_0^2 + r^2}{2br}. \tag{14}$$

On a surface of revolution, the geodesic curvature of a line crossing a meridian under an angle α is

$$\kappa_u = \frac{r'}{rA}\sin\alpha + \frac{\alpha'}{A}\cos\alpha. \tag{15}$$

Upon substituting (8), this expression and its counterpart for the v-line acquire the form

$$\kappa_u = -\frac{r'}{rA}\frac{\sin\omega}{\cos\beta}, \qquad \kappa_v = \frac{r'}{rA}\frac{\sin\omega}{\cos\alpha}. \tag{16}$$

Formulas (5), (9) and (16) are sufficient for solving the system (1) with respect to the net forces and one of the geometric parameters. In fact, the forces could be obtained on the basis of formulas (9) alone using the global equilibrium conditions of the net, whereupon it would be possible to address the geometric aspect of the problem. However, the objective of this study is not just to solve the problem but also to reveal and investigate the statical-geometric interrelation. This is best accomplished by examining the static vector (3).

CHEBYSHEV NETS: STATICS

By virtue of axial symmetry the second component of the static vector must vanish. After setting $k = 2$ in (3) and rearranging it using appropriate substitutions, the following equation is obtained:

$$r'\left(\frac{a\sigma_u}{b\sigma_v}-\frac{b\sigma_v}{a\sigma_u}\right)-r\left(\frac{\sigma_u'}{\sigma_u}-\frac{\sigma_v'}{\sigma_v}\right)\cos\alpha\cos\beta=0. \tag{17}$$

Taking into account (8), the solution of this equation is found to be

$$\frac{\sigma_u}{\sigma_v}=\frac{b}{a}\frac{\sin\alpha+C_1\sin\beta}{C_1\sin\alpha+\sin\beta}, \tag{18}$$

where C_1 is an arbitrary constant. Of all axisymmetric Chebyshev nets, those and only those satisfying relation (18) are static.

Employing this relation allows the first component of the vector (3) to be presented as

$$S_1=S'=\frac{\beta'}{\sin\alpha\cos\beta}\frac{\sin\alpha+C_1\sin\beta}{C_1\sin\alpha+\sin\beta}-\frac{\sigma_v'}{\sigma_v}+\frac{\omega'\cos\omega}{\sin\omega}, \tag{19}$$

wherefrom the static potential is

$$S=\ln\frac{C_1\sin\alpha+\sin\beta}{\sigma_v\cos\beta\sin\omega}, \tag{20}$$

and the net forces (4) are

$$T_u=C\frac{C_1\sin\alpha+\sin\beta}{\cos\alpha\sin\omega},\qquad T_v=-C\frac{C_1\sin\beta+\sin\alpha}{\cos\beta\sin\omega}. \tag{21}$$

To evaluate the constants C and C_1 consider the equilibrium of a net element (Fig. 2) relating the above net forces to the meridional, T_1, and shearing, T_{12}, forces per unit length of a parallel:

$$\begin{aligned}T_1\,ds_2&=T_u\,ds_v\cos\alpha+T_v\,ds_u\cos\beta,\\ T_{12}\,ds_2&=T_u\,ds_v\sin\alpha+T_v\,ds_u\sin\beta.\end{aligned} \tag{22}$$

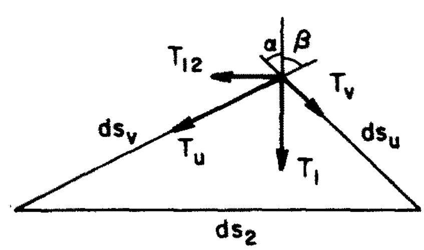

Fig. 2. Net forces and their components.

As is readily seen from Fig. 2,

$$\frac{ds_2}{\sin \omega} = \frac{ds_u}{\cos \beta} = \frac{ds_v}{\cos \alpha}, \tag{23}$$

so that

$$T_1 = \frac{1}{\sin \omega}(T_u \cos^2 \alpha + T_v \cos^2 \beta),$$

$$T_{12} = \frac{1}{\sin \omega}(T_u \sin \alpha \cos \alpha + T_v \sin \beta \cos \beta). \tag{24}$$

These forces result, respectively, in the axial force, T_z, and the torque moment, M_z:

$$T_z = 2\pi r T_1 \sin \theta, \qquad M_z = 2\pi r^2 T_{12}, \tag{25}$$

where θ is the angle between the normal to the surface and the axis of revolution, or the meridian slope (Fig. 3).

Combining eqns (21), (24), (25) and (12) enables the arbitrary constants to be expressed in terms of T_z and M_z:

$$CC_1 = \frac{M_z}{2\pi r_0^2}, \tag{26}$$

$$C = \frac{\sin \omega}{2\pi r}\left[\frac{T_z}{\sin \theta} - \frac{M_z(b^2 \tan \alpha + a^2 \tan \beta)}{r r_0^2}\right], \tag{27}$$

which, upon substitution into (21), yield

$$T_u = \frac{1}{2\pi r}\left(\frac{T_z}{\sin \theta}\frac{\sin \beta}{\cos \alpha} - \frac{aM_z}{br}\right), \qquad T_v = \frac{1}{2\pi r}\left(\frac{T_z}{\sin \theta}\frac{\sin \alpha}{\cos \beta} - \frac{bM_z}{ar}\right). \tag{28}$$

The global forces T_z and M_z are either the external loads applied to the net edges or the constraint reactions of the prestressed net with fixed edges.

PROPERTIES AND CONFIGURATIONS OF A TORQUE-FREE NET

An important by-product of the above calculation is relation (27) which can be rewritten this way:

$$\frac{\sin \omega}{r \sin \theta}\left(1 - \frac{M_z}{T_z}\frac{\sin \theta}{r}\frac{b^2 \tan \alpha + a^2 \tan \beta}{r_0^2}\right) = \frac{2\pi C}{T_z} = \text{const.} \tag{29}$$

It establishes a general interrelation between the statical and geometric parameters of an

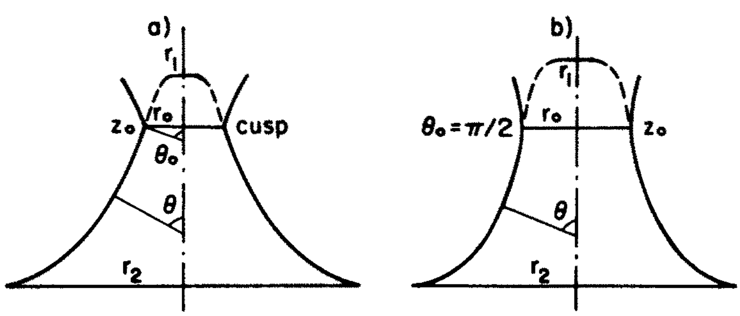

Fig. 3. Profiles of torque-free Chebyshev nets.

axisymmetric Chebyshev net. The most interesting and rather peculiar case is a torque-free net. Setting $M_z = 0$ and taking advantage of (9) reduces (29) to any of the following:

$$\sin\theta = \frac{r_0 \sin\theta_0}{\sin\omega_0}\frac{\sin\omega}{r} = \frac{\sin\theta_0}{\cos\alpha_0}\cos\alpha = \frac{\sin\theta_0}{\cos\beta_0}\cos\beta, \tag{30}$$

where the subscript 0 indicates the reference parallel z_0, i.e. that with $r = r_0$. The forces in this net are found from either (21) or (28):

$$T_u = \frac{C\sin\beta}{\cos\alpha\sin\omega} = \frac{T_z}{2\pi r}\frac{\sin\beta}{\sin\theta\cos\alpha}, \qquad T_v = \frac{C\sin\alpha}{\cos\beta\sin\omega} = -\frac{T_z}{2\pi r}\frac{\sin\alpha}{\sin\theta\cos\beta}, \tag{31}$$

and the force ratio is

$$\frac{T_u}{T_v} = -\frac{a\sin\beta}{b\sin\alpha}. \tag{32}$$

Thus, in spite of the fact that the net is skewed (twisted) and lacks symmetry with respect to a meridional plane, it is torque-free; as is seen from (24) and (31), the shearing components of the net forces cancel each other.

The shape of the net is easily determined by a forward integration procedure of Euler from the equation

$$dz = \tan\theta\, dr \tag{33}$$

after $\tan\theta$ is expressed in terms of r using one of the relations (30) and (14). The integration can be done upon assigning an appropriate set of four arbitrary parameters. At least one of the parameters must be a linear dimension (e.g. r_0, a or b) thus determining the actual size of the net or serving as a size scale factor. The remaining parameters can be other dimensions, slopes (θ), angles (α and β), force ratios (T_u/T_v), etc. These may be assigned at one or more locations along the z-axis (parallels) although a mathematically correct and computationally convenient assignment involves one location. A natural choice for this location is $z = z_0$ where, by virtue of (13), (18) and (31),

$$\alpha_0 = 0, \qquad \beta_0 = \arccos\frac{a}{b}, \qquad \sigma_u^{\ 0} = 0, \qquad T_v^{\ 0} = 0, \qquad T_u^{\ 0} = \frac{T_z}{2\pi b\sin\theta_0}. \tag{34}$$

It can be found from eqns (30) and (33) that

$$r_0'' = r''|_{\alpha=0} = 0, \tag{35}$$

i.e. (r_0, z_0) is an inflection point on the surface meridian, and the surface segment above the reference parallel $z = z_0$ has positive Gaussian curvature. Here the net cannot be prestressed since one of the forces would have to be compressive as is also seen from (32) at $\alpha > 0$. In order to extend the net beyond $z = z_0$, the mirror image of the lower segment of the surface can be used. Generally, this will give rise to a cusp on the surface meridian (as is shown in Fig. 3a) in which case a structural ring is needed to support the radial resultant of the net forces, $T_r^0 = T_u^0\cos\theta$. However, in the particular case of $\theta_0 = \pi/2$, the meridian is smooth (Fig. 3b) and, according to (30) and (34),

$$\sin\theta = \cos\alpha. \tag{36}$$

Note that $z = z_0$ is the plane of symmetry of the net surface but not necessarily of the net itself. It is both geometrically and statically possible that the net changes the direction of twist when passing through $z = z_0$, i.e. α and β either retain their signs or change them simultaneously.

At both the planes $z = z(r_1)$ and $z = z(r_2)$, the meridian slope, θ, is 0 so that both T_u and T_v approach infinity if $T_z \neq 0$ and, obviously, the adjacent segment of the net cannot support prestress.

ORTHOGONAL STATIC NETS

Orthogonal nets on any surface are determined to within one arbitrary function of two variables. Actually, this function generates a one-parametric array of lines which can always be supplemented with orthogonal trajectories to form a net. Hence, there exist "many more" orthogonal nets on a surface than Chebyshev nets. An axisymmetric orthogonal net is conveniently defined by the angle, γ, that the first array of lines forms with a meridian (Fig. 4). The question is, what must this angle be in order for the net to be static.

One approach to the problem would be to take advantage of an analogy between an orthogonal cable net and a net of the principal stress trajectories of a membrane. An alternative approach used here is to investigate again the static vector (3) which is simplified considerably due to the fact that the net angle, ω, is $\pi/2$ and the geodesic and Chebyshev curvatures of the orthogonal net lines coincide. Hence, in this case

$$S_k = H\left(\frac{\kappa_u}{\sigma_u}u_k + \frac{\kappa_v}{\sigma_v}v_k\right) - \frac{\sigma_{u,i}}{\sigma_u}v^i u_k + \frac{\sigma_{v,i}}{\sigma_v}u^i v_k, \tag{37}$$

where

$$H = \frac{1}{2}(\sigma_u + \sigma_v) = \frac{1}{2}(\sigma_1 + \sigma_2) \tag{38}$$

is the mean curvature of the surface, σ_1 and σ_2 are its principal curvatures.

Letting $k = 2$ and using expressions (5), (15) and Euler's formulas

$$\sigma_u = \sigma_1 \cos^2\gamma + \sigma_2 \sin^2\gamma, \qquad \sigma_v = \sigma_1 \sin^2\gamma + \sigma_2 \cos^2\gamma \tag{39}$$

allows S_2 to be transformed into the following final form

$$S_2 = \frac{B}{2A\sigma_u\sigma_v}[\sin 4\gamma(H'\sigma_2 - 2H\sigma_2') + 4H\sigma_2\gamma'] = 0, \tag{40}$$

wherefrom

$$\tan 2\gamma = \frac{C_1{\sigma_2}^2}{H}. \tag{41}$$

Now, using (41), the first component of the static vector can be rearranged in a similar fashion, yielding

$$S = \ln(\sec 2\gamma \csc^2\theta) \tag{42}$$

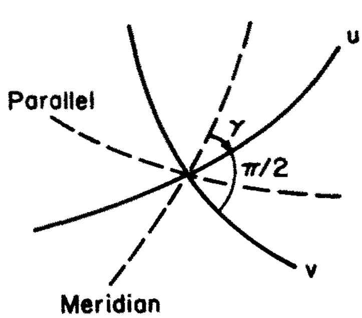

Fig. 4. Element of an orthogonal net.

and, according to (4),

$$T_u = C\sigma_v \sec 2\gamma \csc^2 \theta, \qquad T_v = -C\sigma_u \sec 2\gamma \csc^2 \theta. \tag{43}$$

Evaluating the arbitrary constants C and C_1 in terms of T_z and M_z results in expressions for the net forces recognizable as the principal forces in membrane shell theory. It is by far more interesting to observe the dissimilarity between an orthogonal static net, on the one hand, and a Chebyshev or a geodesic static net[8] (which are rather similar) on the other.

First, as a result of orthogonal nets being "more numerous" orthogonal static nets exist on every surface, which was not the case with either Chebyshev or geodesic nets. Second, on a given surface of revolution, orthogonal static nets are determined to within one arbitrary constant, C_1 in (41); those and only those nets satisfying this relation are static. Third, apart from a trivial case of the net of meridians and parallels, torque-free orthogonal nets exist on only one surface of revolution. Indeed, the shearing force, on the basis of (24), (43) and (38), is

$$T_{12} = (T_u - T_v)\sin 2\gamma = CH \csc^2 \theta \tan 2\gamma \tag{44}$$

and, except for the mentioned trivial case of $\tan 2\gamma = 0$, T_{12} vanishes only on surfaces with zero mean curvature H (minimal surfaces). The only minimal surface of revolution is a catenoid. All of its orthogonal nets are static and are characterized by $\sigma_u = -\sigma_v$ so that the two net forces (43) are equal to each other. However, in the presense of torque the two forces cannot be equal and equilibrium is possible only for the asymptotic net of the catenoid.

ASYMPTOTIC NETS

Asymptotic nets are those with $\sigma_u = \sigma_v = 0$. They exist only on surfaces of negative Gaussian curvature and are different in principle from other nets. Since the third of the equilibrium conditions (1) is satisfied identically, the remaining two always allow a non-trivial solution so it can be stated that asymptotic nets are static. On a surface of revolution, the asymptotic net is symmetric relative to a meridional plane, i.e. it possesses both rotation and reflection symmetry. As a result,

$$\alpha = -\beta, \qquad \omega = 2\beta, \qquad \kappa_u = -\kappa_v, \qquad \lambda_u = -\lambda_v. \tag{45}$$

Employing these relations as well as (5), (7) and (15) and looking first for a symmetric solution, $T_u = T_v = \bar{T}$, enables the first of the eqns (1) to be written as

$$\bar{T}' \frac{\cos\beta}{A} \sin 2\beta - \frac{2\bar{T}}{A}\left(\beta' \cos\beta - \frac{r'}{r}\sin\beta \cos 2\beta\right) = 0. \tag{46}$$

The solution is

$$\bar{T} = C\frac{\tan\beta}{r\sin\theta}, \tag{47}$$

or, upon evaluating the arbitrary constant C in terms of T_z,

$$T_u = T_v = \frac{T_z \tan \omega/2}{2\pi r \sin\theta}. \tag{48}$$

In the same way an antisymmetric solution, $T_u = -T_v = \tilde{T}$, is found. The first equation of system (1) becomes

$$\tilde{T}' \frac{\cos\beta}{A} \sin 2\beta - \frac{2\tilde{T}}{A}\frac{r'}{r} 2\sin\beta\cos^2\beta = 0, \tag{49}$$

wherefrom

$$\bar{T} = C_1/r^2 \tag{50}$$

or

$$T_u = -T_v = M_z/2\pi r^2. \tag{51}$$

Note that for this case, according to (25),

$$\bar{T} = T_{12} \tag{52}$$

i.e. the net force produced by torsion coincides with the shearing force.

Combining solutions (48) and (51) gives

$$T_u = \frac{1}{2\pi r}\left(\frac{T_z \tan \omega/2}{\sin \theta} + \frac{M_z}{r}\right), \qquad T_v = \frac{1}{2\pi r}\left(\frac{T_z \tan \omega/2}{\sin \theta} - \frac{M_z}{r}\right). \tag{53}$$

Of course, this statically possible state can be realized (e.g. as a prestress) only when both the forces are tensile.

The above results show explicitly that the asymptotic net of a surface of revolution is static. What is more important, a converse proposition is also true: if a net with rotation and reflection symmetry is in equilibrium under axial tension and torque, it is the asymptotic net of some surface of revolution.

The proof is elementary. The stipulated type of the net and load symmetry entails

$$\sigma_u = \sigma_v = \sigma, \qquad T_u = \bar{T} + \bar{T}, \qquad T_v = \bar{T} - \bar{T}. \tag{54}$$

But, according to the third of eqns (1) this is only possible when $\sigma = 0$, which was to be proven.

One of the most interesting particular results following from the proposition concerns the Chebyshev net which, as is known, serves as the asymptotic net of a pseudosphere (Fig. 5 [7]). Hence, a segment of a Chebyshev net (e.g. a basketball net) stretched between two parallel rings acquires the form of a hyperbolic, parabolic or elliptic pseudosphere, depending on the boundary values of the angles ω and θ. This curious result is, apparently, unknown in differential geometry which is surprising since a similar problem for a fluid film—the classic Plateau problem—was solved long ago (its solution for this case is a catenoid of revolution).

Asymptotic nets do not have equilibrium loads normal to the surface. When subjected to such a load, the net must undergo finite elastic displacements before it is able to balance the load. This is not necessarily the case with other torque-free static nets which possess some

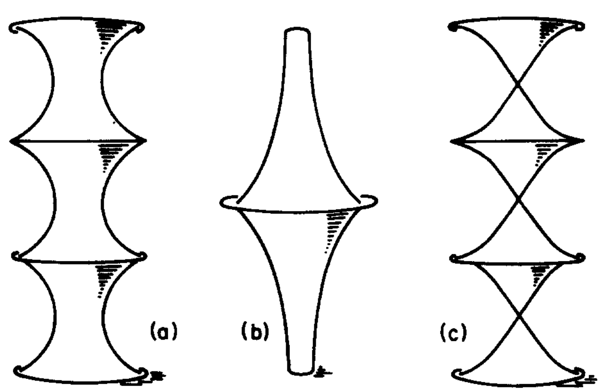

Fig. 5. Pseudospherical surfaces of revolution: (a) hyperbolic; (b) parabolic; (c) elliptic.

normal equilibrium loads. Here, however, even an axisymmetric equilibrium load induces some torsion which must be resisted by the net edge supports.

CONCLUSIONS

With the completion of the investigation of asymptotic nets, all of the major types of axisymmetric static nets have been covered: geodesic[8], Chebyshev, orthogonal and asymptotic. Geodesic and Chebyshev axisymmetric nets exist on every surface of revolution and are determined by two arbitrary constants appearing, respectively, in the Clairaut formulas (see (7) in [8]) and in the formulas (9) of this paper. Only a few of these nets are static and they are found on the corresponding special surfaces of revolution. The most interesting among these are the torque-free nets which were explored in more detail.

Orthogonal axisymmetric nets are determined on any surface of revolution to within one function of one variable. Among them, the static nets are determined only to within one arbitrary constant, but they still are found on any surface. The torque-free nets exist only on a catenoid of revolution where all orthogonal nets are static.

All asymptotic nets (not necessarily axisymmetric) are static. An axisymmetric net with reflection symmetry under a general axisymmetric edge load (tension and torque) is asymptotic and, therefore, static.

The characteristic properties of all the above static nets were revealed and investigated. Most of the statical and geometric attributes of these nets are described by solutions obtained in closed form.

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